Statistical Analysis of the Whole Body Specific Absorption Rate using Human Body Characteristics

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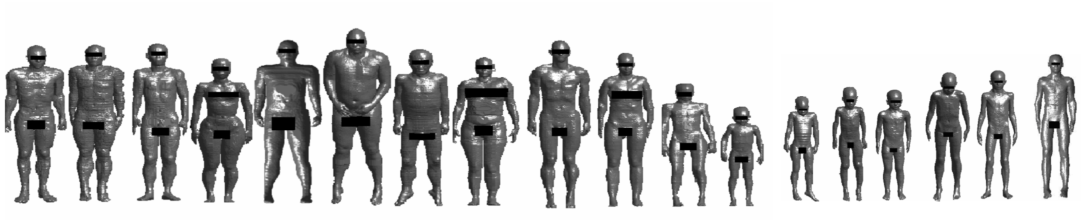
Context

- Normative context
  - Basic Restrictions (BR):
    - SAR over 10 grams and over the whole body
    - Difficult to check in situ
  - Reference Levels (RL) :
    - max. allowed EMF and power density
    - Guaranty the compliance to BR
    - Established several years ago

- Dosimetric context
  - Anatomical phantoms
  - Large variability of the exposure in the existing set of phantoms

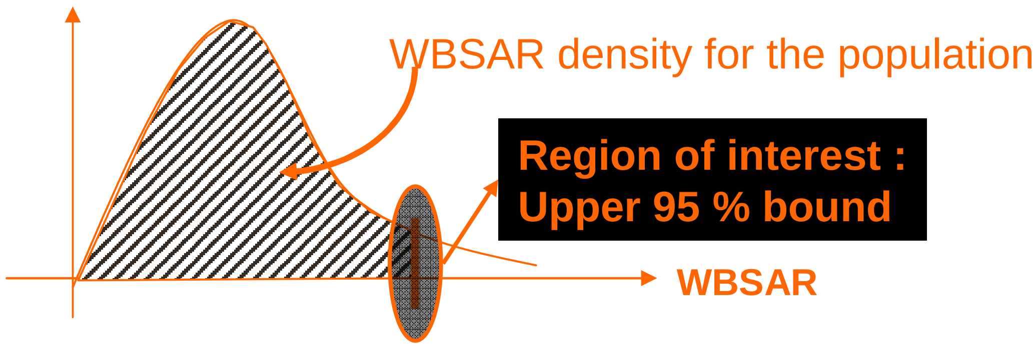


How to characterize the exposure for an entire population ?

Problematic : How to characterize the exposure for an entire population ?

- 18 phantoms ~~➔~~ Monte Carlo method

- WBSAR surrogate model
  - Focus on the probability of failure.



- Exposure configuration : frontal plane wave polarized vertically at the frequency 2100 MHz and incident power of 1W/m²

How to build such surrogate model ?

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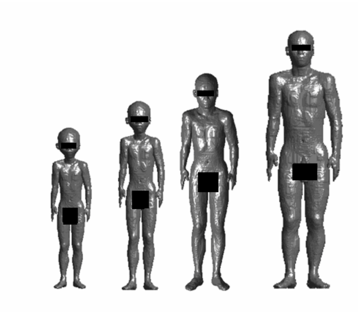
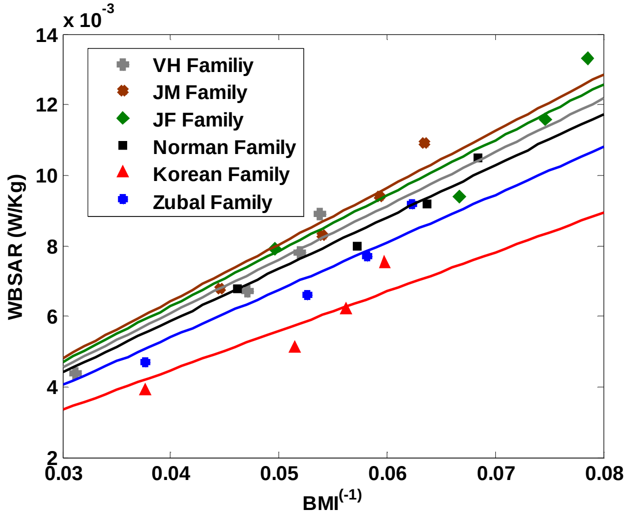
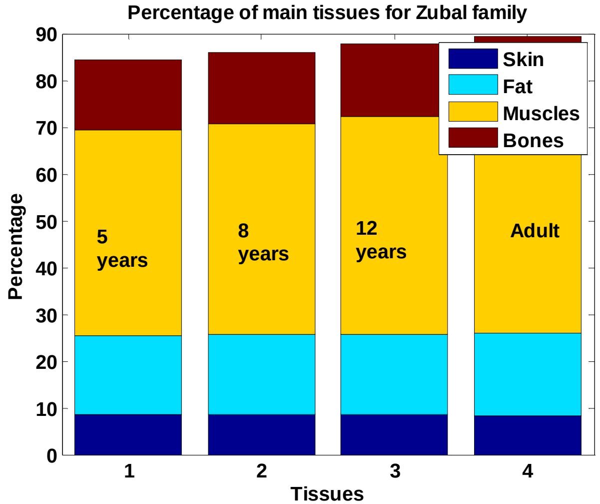
Surrogate model building

Observation on phantom's families



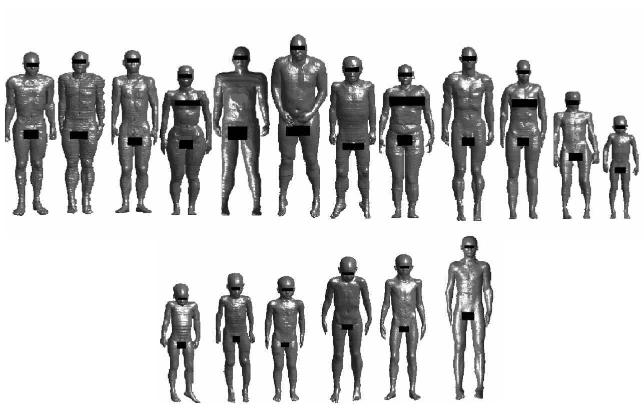




WBSAR calculated using the FDTD method

$$y = \beta X + \varepsilon,$$

- y → WBSAR
- β → unknown parameter depending on internal morphology
- X → inverse of the Body Mass Index (BMI =weight/height²)
- ε → estimation error



β estimated by Least square Method → 30% of error on the WBSAR

Description of the surrogate model

- $Y = \beta.X$
- Input data
  - X = inverse of BMI → statistical available data in literature
  - β = internal morphology → no statistical data for populations
    - Unknown statistical distribution :
    - Estimation of the distribution by different types of statistical laws (parametric ones and Gaussian mixture)
- Output data
  - Threshold of WBSAR at 95%

What is our knowledge on β?

Knowledge on β

Physical knowledge

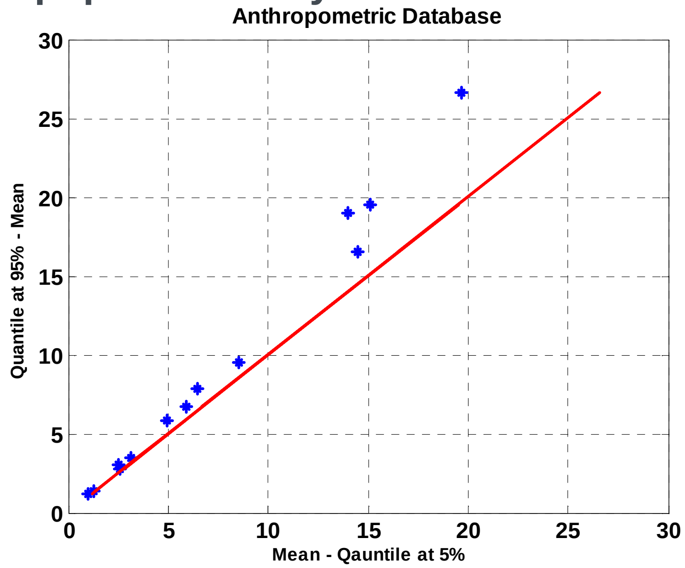
- β depends on internal morphology
- β is bounded (human morphology is bounded) : β_{low} and β_{upp} (lower and upper bound)
- β is positive (WBSAR is positive)

Additional constraints

- Mean value : most of phantoms match to the mean of population they come from

$$<\beta>=\frac{\sum_{i=1}^{18}\beta_i}{18}\approx 0.15$$

  - β_{low} = 0 (WBSAR >0)
  - β_{upp} = 0.3: chosen by assuming a symmetry around the mean

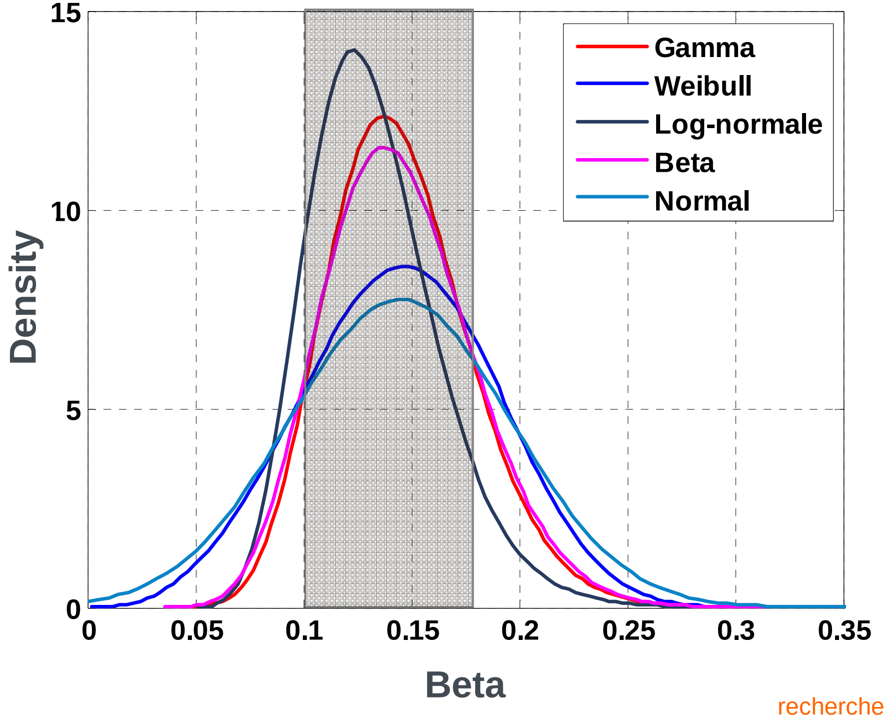


- The independence between β and X (to be relaxed)

Parametric laws : Gamma, Beta, Normal, Weibull and Log-normal

Most of morphological external factors belongs to the parametric laws family.

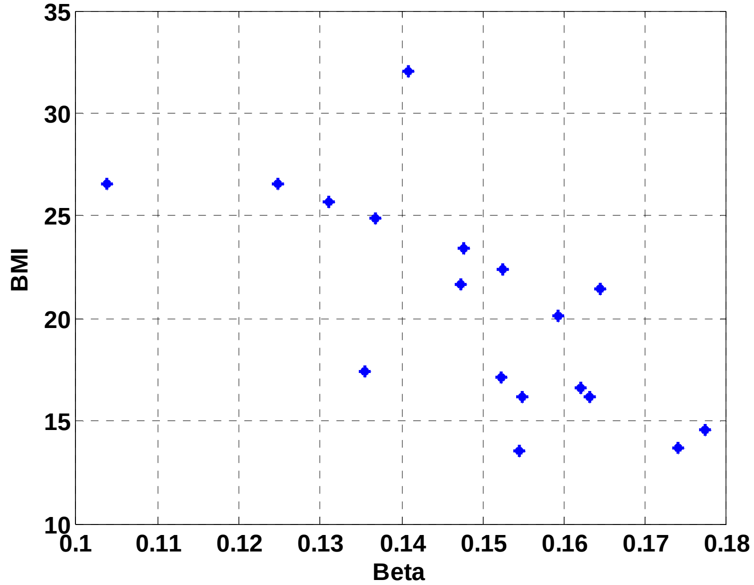
Application to the French population aged of 20 years : BMI~N(22.29, 2.9^2)



Param. laws	Threshold of the WBSAR at 95 % (mW/Kg)
Gamma	9.2
Beta	9.3
Normal	11
Weibull	10.5
Log-normal	9.1
Std/mean	9%

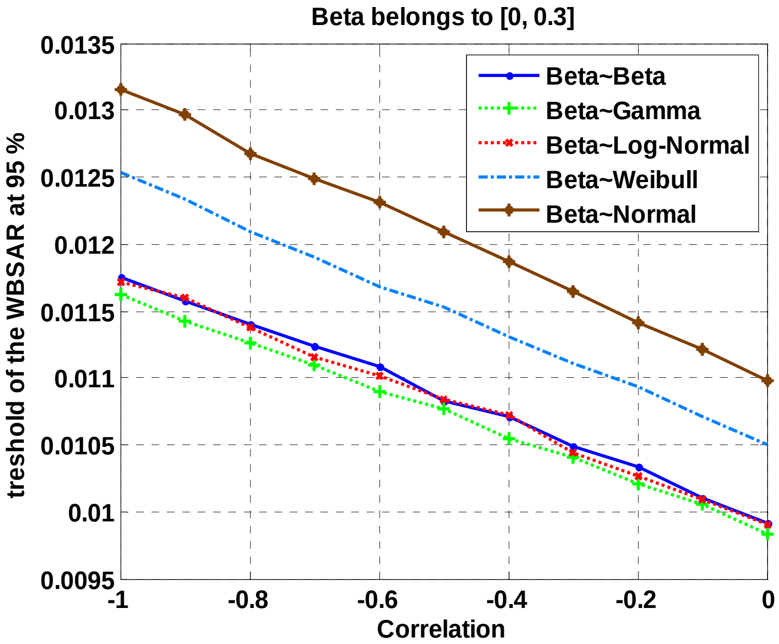
Weak variability of the WBSAR at 95% whatever the parametric law

Relaxation of the independency between β and X



→ negative correlation

Relaxation of the independency



correlation ↗ → WBSAR at 95% ↗

Gaussian Mixture

- **Density probability function for β that maximizes the threshold of the WBSAR.**

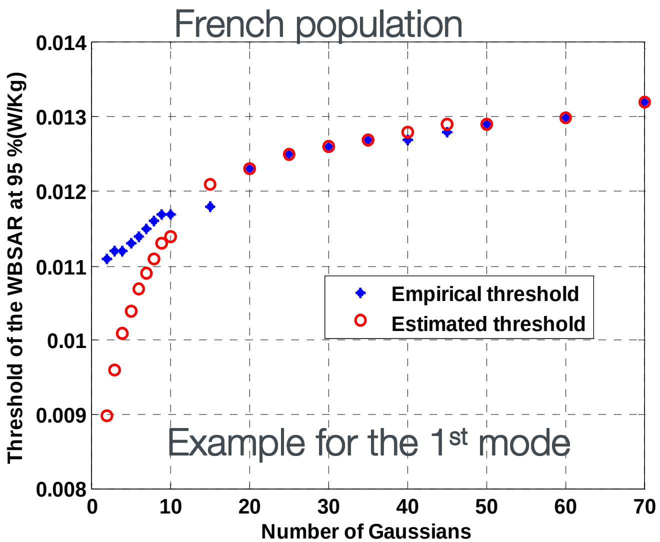
$$\begin{cases} p_\beta(\beta) = \sum_{i=1}^{n} p_i g_{m_i,\sigma_i}(\beta) \\ \sum_{i=1}^{n} p_i = 1 \\ 0 \le p_i \le 1 \end{cases}$$

- **From knowledge to Constraints :**
  - **β positive and belongs to $[\beta_{low}, \beta_{upp}]$: $m_n + 3\sigma_n = \beta_{upp}$ and $m_1 - 3\sigma_1 = \beta_{low}$**
    → 99.74% of the probability density belongs to $[\beta_{low}, \beta_{upp}]$

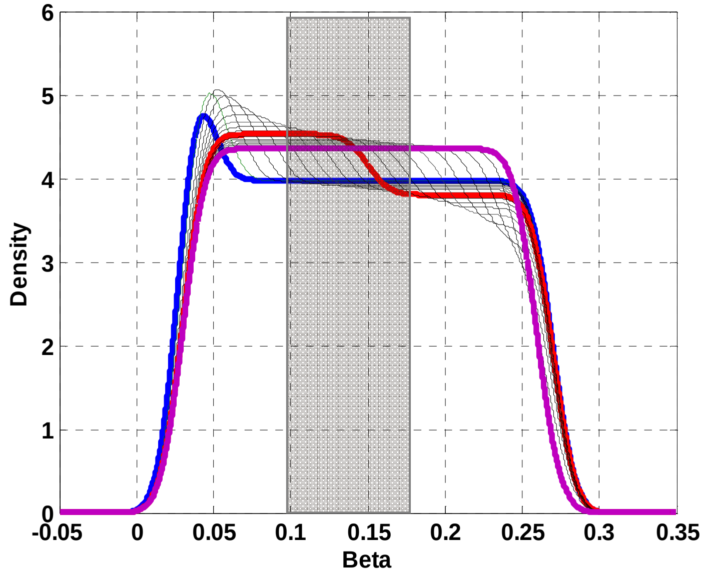
- **Additional constraints :**
  - **Known mean :** $\sum_{i=1}^{n} p_i m_i \approx 0.15$
  - **σ_i does not appear in the estimated CDF : choice of $\sigma_i = \sigma$**
  - **Smoothness : $m_i - m_{i-1} = \sigma_i$ (Rayleigh Criterion)**
  - **Unimodality : PDF with single mode ($p_i \leqq p_{i+1} \leqq \ldots \leqq p_j$ et $p_j \geqq p_{j+1} \geqq \ldots \geqq p_n$)**

Number of Gaussian in mixture

- Taylor development → CDF WBSAR
  - estimated threshold by approximation of the CDF compared to the empirical threshold obtained using Monte Carlo



PDF obtained for 20 Gaussian



	Gamma	Beta	Normal	Weibull	Log-normal	Gaussian mixture
Threshold of the WBSAR at 95 % (mW/Kg)	9.2	9.3	11	10.5	9.1	**13.2**

- 1st mode
- median mode
- last mode

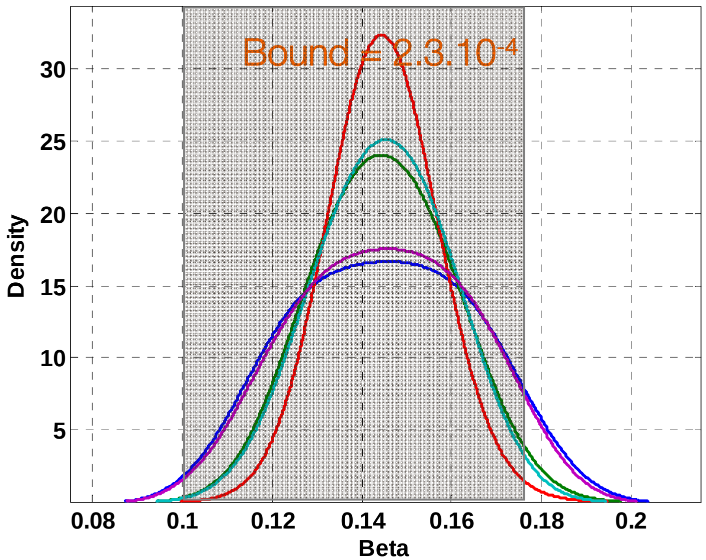
Bell-shaped Gaussian mixture

To obtain bell-shaped PDF, a constraint on the variance is introduced :

$$\mathrm{var}(\beta) = \sum_{i=1}^{n} p_i (m_i^2 - 2 < \beta > m_i) + \sigma^2 + < \beta >^2 \leq \mathrm{Bound}$$

The initial variance depends on the mode and the number of Gaussians:

For a mixture of 20 Gaussians the variance belongs to [$4.5.10^{-3}$,$5.3.10^{-3}$]



	Threshold of the WBSAR at 95 % (mW/Kg)
Gamma	9.2
Beta	9.3
Normal	11
Weibull	10.5
Log-normal	9.1
Gaussian mixture	**8.9**

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Context

- How to separate the internal and external factors impacting the WBSAR?

  → use of homogeneous phantoms

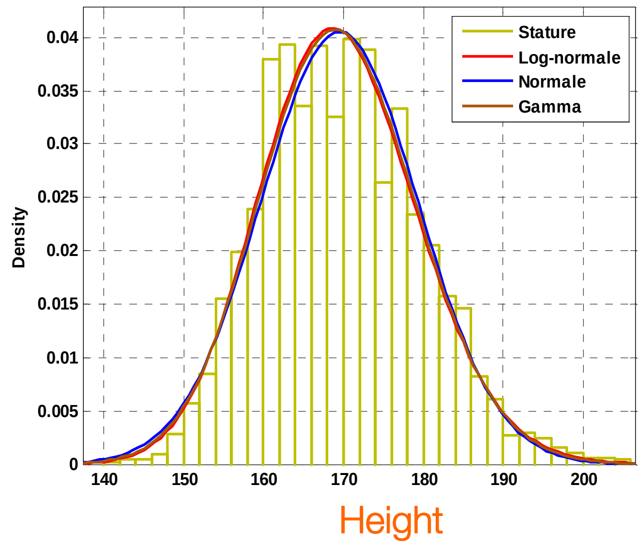
- Homogeneous phantoms
  - Equivalent liquid (IEC)
  - Morphing technique to deform the external shapes of the existing phantoms

- Monte Carlo is still expensive to characterize the threshold of the WBSAR at 95 % for a given population.
  - Sequential Bayesian Experiment Planning : allow finding the morphology corresponding to the region of 95 %.

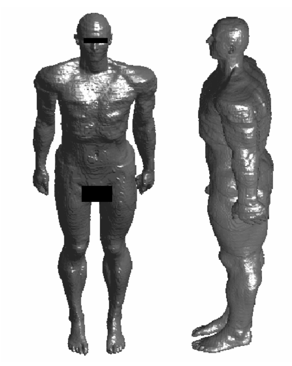
How to extend the database of phantoms?

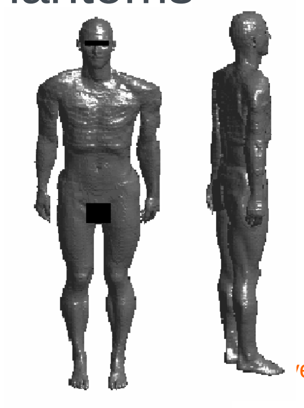
- Morphing technique to deform homogeneous phantoms
  - Deforming factors used: height, inside leg height, front shoulder breadth, chest and waist
  - Laws of the deforming factors: parametric laws

- Studied population
  - Anthropometric database of French population
  - Sample of 3800 adults
  - Log-normal and normal laws to estimate the density of the deforming factors



- Examples of morphed phantoms





Parametric model

- Using existing simulations, a suitable model is written as follow

$$\text{WBSAR} = \theta_1 \text{Height} + \theta_2 \frac{\text{chest}}{\text{front shoulder breadth}} + \theta_3 \frac{\text{waist}}{\text{front shoulder breadth}} + \theta_4 + \varepsilon$$

- Initial observations
  - D-optimal design to choose observations that allow obtaining a small confident region for the parameters
  - 6 initial observations $F_n = (x_i, y_i)_{i=1,\dots,n}$
    - Xi = factors of the parametric model (height, waist…)
    - Yi = WBSAR calculated with FDTD

Height	F.S.B	chest	waist	WBSAR (mW/kg)
1,57	0,346	1,144	1,056	3,87
1,81	0,417	1,246	0,899	4,7
1,918	0,389	1,147	1,197	4,48
2,02	0,524	1,114	0,896	5,68
1,98	0,435	0,968	1,024	4,75
1,38	0,372	0,782	0,663	7,64

Choose observations to refine the region of interest (WBSAR at 95%)

Sequential Bayesian Experiment Planning

initial observations $F_n = (x_i, y_i)_{i=1,...,n}$ ➜ **Bayes Formula** $P(\Theta \setminus F_n) = P(\Theta).P(F_n \setminus \Theta)$

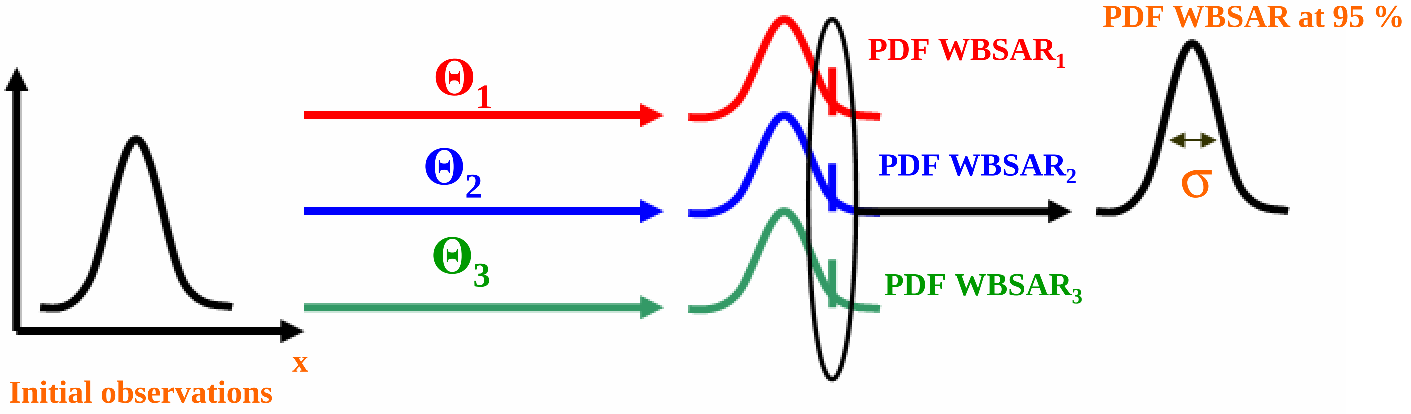
Prior Law : Non-informative

Posterior law

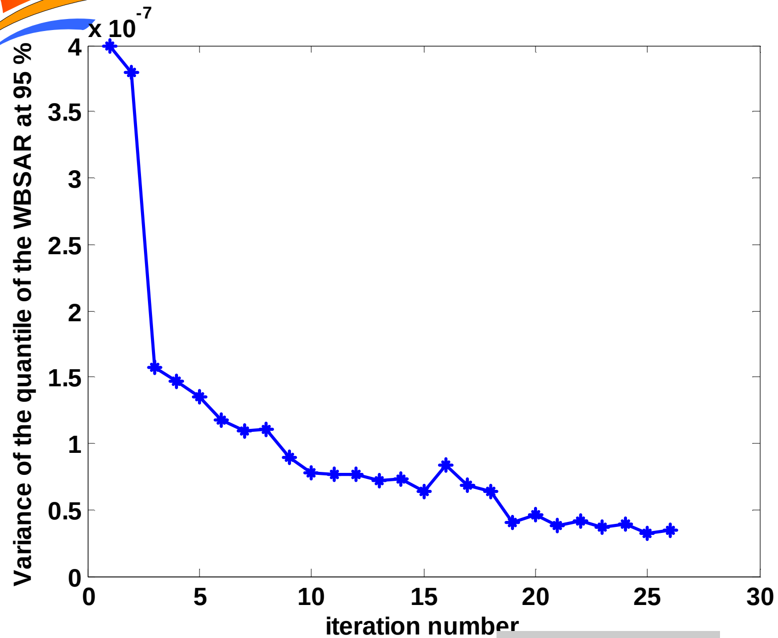
Likelihood

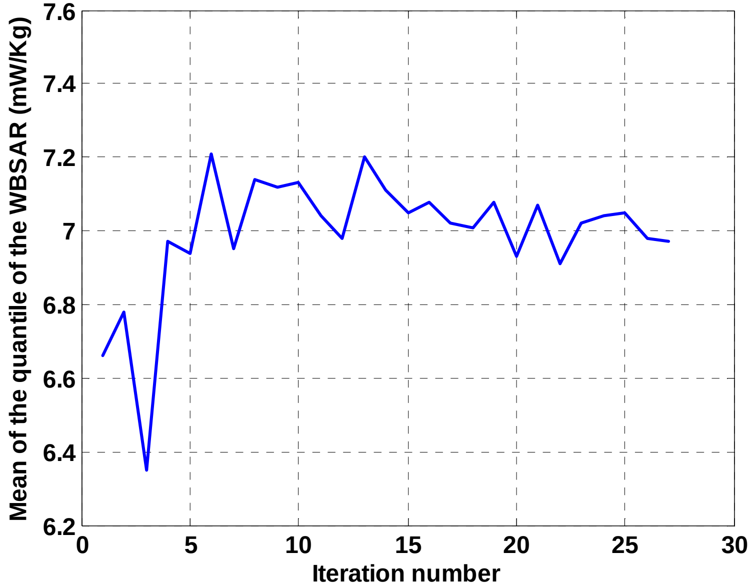
where $\Theta = [\theta_1, \theta_2, \theta_3, \theta_4]$

Criteria to choose the next candidate: diminution of the variance of the threshold of the WBSAR at 95 %



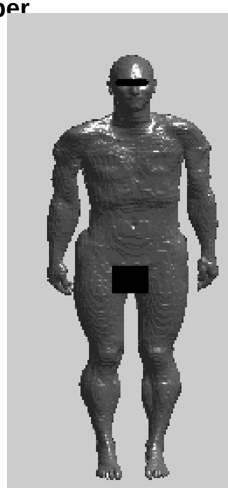
Results





Stability of the mean of the quantile at 0.007 W/Kg

One phantom having a WBSAR of 0,007 W/kg



Height =147 cm
Chest =76.5 cm
Waist = 65 cm
Frontal shoulder breadth= 34 cm
Weight = 49 Kg

WBSAR = 0,007W/kg

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Conclusion

- WBSAR surrogate model for **heterogeneous** phantoms
  - External morphology characterized by the BMI
  - Internal morphology described by different types of statistical laws
    - → weak variability of the WBSAR at 95% in term of the type of the laws
    - →WBSAR at 95% maximized using Gaussian mixture (0,013W/kg)

- WBSAR parametric model for **homogeneous** phantoms
  - Morphing technique to obtain additional phantoms
  - Baysian inference to choose new observations
    - →after 25 iterations, stability of the variance of the WBSAR at 95%
    - →0,007 W/kg: mean value obtained for the WBSAR at 95%

- Future work : to assess the influence of the dielectric properties in the parametric model

thank you